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HYDRODYNAMIC PHENOMENA DURING INTERACTION OF OPTICAL RADIATION WITH STRONGLY ABSORPTIVE DIELECTRIC FLUIDS

V. V. Zosimov, M. Yu. Kukushkin, K. A. Naugol'nykh, and O. V. Puchenkov

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At this time various aspects of intense optical radiation interaction with strongly absorptive fluids are considered in numerous experimental and theoretical research studies [1-7]. The traditional experimental method of investigating this interaction is to record the acoustic perturbations occurring under the action of the optical radiation in the fluid and the gas adjoining it. Such recording is performed by pressure sensors of different constructions [3, 6] as well as by using optical shadow methods [1, 2, 5]. However, because there is no generally recognized theoretical model of the process of intensive optical radiation interaction with strongly absorptive fluids, experiments associated with acoustic measurements do not receive a single interpretation in the research articles of different authors [4, 6, 7]. Consequently, there is the need to obtain new additional data about the physics of the occurring phenomena.

The hydrodynamic perturbations of the interfacial liquid—gas boundary accompanying the interaction process are investigated in this paper. The time scale for development of these perturbations considerably exceeds the characteristic time of acoustic perturbation formation and evolution. This ("hydrodynamic") stage of the interaction process has not been studied in detail (the experiments [8] are of qualitative, demonstrative nature). However, there are papers devoted to a study of similar hydrodynamic phenomena in a fluid caused by by an explosion on its surface [9, 10]. The investigation we performed permitted setting up a linear relation between the total recoil pressure pulse of the pair escaping from the surface and acting on the fluid surface and the energy density of the incident laser radiation in a certain range of laser radiation intensity. The proportionality factor within the limits of measurement accuracy is identical for the three liquids under consideration (water, ethanol, heated glycerine).

1. FORMULATION OF THE PROBLEM

Absorption of sufficiently high laser radiation energy in a fluid is accompanied by intensive vapor-formation processes in its near-surface layer [6]. A certain pressure acts on the surface because of the recoil pulse of the vapor escaping from the surface, resulting in the generation of sound and causing a residual flow in the liquid half-space. Neglecting viscosity effects as well as the entropy change, an integral relationship

$$\varphi(\tau) = -\frac{1}{\rho_0}\int_0^\tau p'\,dt + \frac{1}{\rho_0}\int_0^\tau \frac{p'^2}{\rho_0 c^2}\,dt - \int_0^\tau \frac{v^2}{2}\,dt, \tag{1.1}$$

can be written for the potential of this flow, where ρ_0 and c are the density and sound speed, p' is the excess pressure occurring under the action of the recoil pulse, v is the motion velocity, and τ is the duration of the recoil pulse. Considering the case of moder-

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ate pressures when $p' \ll \rho_0 c^2$, we neglect the second integral in (1.1). The excess pressure p' on the free surface equals the external pressure pulse P. Letting a denote the characteristic dimension of the domain of pressure pulse action, the fluid velocity near the surface can be estimated as $v \sim \varphi/a$. If just the contribution of the first integral from (1.1) is taken into account in the quantity φ, then we find $v \simeq \Pi/(\rho_0 a)$ $\left(\Pi = \int_0^\tau P dt \text{ is the total recoil pressure pulse}\right)$. Then the last integral from (1.1) is estimated by the expression $\int_0^\tau \frac{v^2}{2} dt \sim \frac{\Pi^2}{2\rho_0^2 a^2}\tau$, while the surface displacement from the equilibrium position up to the time $t = \tau$ has the form $h \sim (\Pi/(\rho_0 a))\tau$. It follows from the estimates presented that upon compliance with the condition

$$\Pi \ll \rho_0 a^2/\tau \tag{1.2}$$

the last integral in (1.1) can be neglected and the fluid surface can also be considered as a plane until termination of the action of the pressure pulse. For the case we examined the interaction between electro-discharge CO_2 laser radiation and fluids τ does not exceed 10 μsec [6, 7]. Considering $a \sim 2\cdot 10^{-3}$ m and $\rho_0 \sim 10^3$ kg/m^3, we obtain the constraint $\Pi \ll 400$ Pa·sec from (1.2).

Therefore, the problem of describing the hydrodynamic perturbations of a fluid surface that occur under the action of powerful optical radiation reduces to the solution of an incompressible flow problem with a given initial potential distribution on a plane free surface

$$\varphi|_{t=0}(\mathbf{r}) = -\frac{1}{\rho_0}\Pi(\mathbf{r}) = -\frac{1}{\rho_0}\int_0^\tau P(t, \mathbf{r})\, dt.$$

2. THEORETICAL ANALYSIS

The solution of the formulated problem is found easily in a linear small-perturbation approximation in the form of gravitational-capillary wave (GCW) expansions. For an axially-symmetric distribution $\Pi(\mathbf{r}) = \Pi_0 f(|\mathbf{r}|)$ it is convenient to take such an expansion by using a Laplace transform in the time variable and a Hankel transform in the space variable [11]. Taking account of the influence of viscosity on the nature of the surface perturbations occurring when the viscosity is sufficiently small: $\nu \ll \omega(k)/(2k^2)$ (ν is the kinematic viscosity, $\omega(k)$ and k are the frequency and wave number connected by the dispersion relation for the GCW spectrum) is by the method indicated in [12]. Under the assumptions made we obtain an expression for the vertical displacements of the surface

$$h(t, r) = \frac{1}{\rho_0}\int_0^\infty \Pi^+(k) k^2 J_0(kr) \frac{\sin(\omega t)}{\omega} \exp\{-2\nu k^2 t\}\, dk. \tag{2.1}$$

Here $\omega = \sqrt{k(g + \sigma k^2/\rho_0)}$; σ is the coefficient of surface tension, g is the acceleration of gravity, J_0 is a Bessel function, and $\Pi^+(k)$ is the Hankel transform of the distribution $\Pi(r)$ of the total recoil pressure pulse. Taking into account that mainly GCW with a length on the order of the size of the perturbation domain are excited, the characteristic time during which the surface is deflected maximally from the equilibrium position Δt can be estimated: $\Delta t \sim [a/(g + \sigma(\rho_0 a^2))]^{1/2}$. In particular, we find $\Delta t \sim 6$ msec for water with $a = 1.5$ mm [a computation on the basis of (2.1) yields $\Delta t = 4.5$ msec]. The water surface profile at the time $t = 47$ msec, when the GCW being propagated was formed in practice, is displayed in Fig. 1 as calculated from (2.1). The computation is executed for the Gauss distribution $\Pi(r) = \Pi_0 \times \exp(-r^2/a^2)$ ($\Pi_0 = 1$ Pa·sec, $a = 1.5$ mm).

In the reverse limit case of very high viscosity ($\nu \gg \omega/k^2$) the solution can be obtained from the linearized Navier–Stokes equation (the boundary condition should also naturally be written with viscosity taken into account). In this case we have the damped aperiodic solution

$$h(t, r) = \frac{1}{\rho_0}\int_0^\infty \Pi^+(k) k^2 J_0(kr) \frac{\exp[-Qt/(2k\nu)] - \exp(-2\nu k^2 t)}{2\nu k^2 - Q/(2\nu k)}\, dk \tag{2.2}$$

($Q = \omega^2/k$). Both the solutions (2.1) and (2.2) are valid under the condition $|h_{max}| < a$, which imposes a constraint of the pressure pulse Π_0. Appropriate estimates are presented

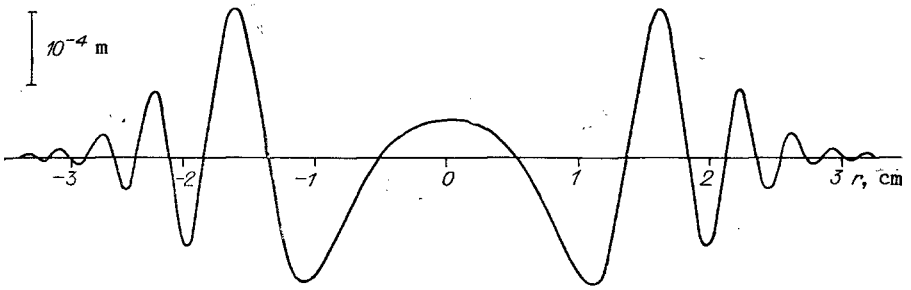

Fig. 1

below in a discussion of the experimental data.

For h > a the problem becomes nonlinear and its complete analytic solution is not obtained successfully. However, a number of experimentally disclosed singularities of the evolution of the surface perturbations being studied permitted development of a theoretical model even for the nonlinear problem. It turns out that in a sufficiently broad range of laser radiation energy interacting with a fluid the deformable section of the surface is a hemisphere expanding within the fluid at a definite stage (see Fig. 3). Such a flow is determined completely by the law R(t) of hemisphere expansion. Let E_0, R_0 be the hemisphere motion energy and radius at the initial time t_0, then the energy balance equation has the form

$$\pi\rho_0 R^3(t)\dot{R}^2(t) + \Delta F_{fr} = E_0, \tag{2.3}$$

where the free energy increment ΔF_{fr} is comprised of the surface energy increment $\Delta F_{sur} = \pi \cdot \sigma(R^2(t) - R_0^2)$ and the potential energy in the gravity field $\Delta F_g(R)$. Under the conditions of our experiment the relationship $\Delta F_{sur} \gtrsim \Delta F_g$ is satisfied for $\sqrt{2\sigma/(g\rho_0)} \gtrsim R$. Integrating (2.3) we obtain

$$t - t_0 = \int_{R_0}^{R} \sqrt{\frac{\pi\rho_0 R^3}{E_0 - \Delta F_{fr}}}\, dR. \tag{2.4}$$

Considering the initial energy E_0 sufficiently large (so that the condition $E_0 \gg \Delta F_{fr}$ will be satisfied for a certain time interval), we write (2.4) in the form

$$R = \left[R_0^{5/2} + \frac{5}{2}\sqrt{\frac{E_0}{\pi\rho_0}}(t - t_0)\right]^{2/5}. \tag{2.5}$$

The initial radius in (2.3)-(2.5) can be considered equal to the characteristic radius of the domain of pressure pulse action $R_0 = a$. In the case of a parabolic total recoil pressure pulse distribution $\Pi(r) = \Pi_0 [1 - (r/a)^2]$, the total flow energy determined from the initial potential distribution is $E_{com} = \frac{16}{15}\frac{\Pi_0^2 a}{\rho_0} \simeq \frac{\Pi_0^2 a}{\rho_0}$. It is easy to see that for

$$\Pi_0 \gg \sqrt{a\sigma\rho_0} \tag{2.6}$$

the fraction of the total energy can be neglected that goes into the formation of a new surface during deformation of the initially plane boundary into the hemisphere of radius a, and it can be considered that $E_0 \simeq E_{com}$ in (2.3)-(2.5). Let us note that if (2.6) is not satisfied, then the whole kinetic energy goes over into surface energy for R $\lesssim a$ and the nonlinear stage of the process does not develop. Therefore, we finally have

$$R(t) = \left[\frac{5}{2\sqrt{\pi}}\frac{\Pi_0}{\rho_0}\sqrt{a}\,(t - t_0) + a^{5/2}\right]^{2/5}. \tag{2.7}$$

The influence of viscosity was not taken into account in the derivation of (2.7). Let us determine the limits of applicability of such an approach. The rate of kinetic energy decrease because of viscosity is given by the expression [12] $\dot{E}_K = -\nu\rho_0 \int \mathrm{grad}\,(v^2)ds$, where the integration is over the fluid surface and the element ds is oriented in the direction of the external normal. Substituting (2.7), we find $\dot{E}_K = -\frac{6\nu}{\rho_0}\frac{\Pi_0^2 a}{R^2}$. The total energy dissipation is $\Delta E = 12\sqrt{\pi}\Pi_0\sqrt{a}\nu\{\sqrt{R} - \sqrt{a}\}$ for hemisphere expansion from an initial radius a to a certain radius R. Therefore, the influence of viscosity can be neglected for $\frac{\Delta E}{E_0} \simeq 12\sqrt{\pi}\frac{\nu\rho_0}{\Pi_0}\sqrt{\frac{R}{a}} \ll 1$.

TABLE 1

Liquid	ρ_0, kg/m^3	σ, N/m	ν, m^2/sec	α, cm^{-1}	λ, kJ/kg	w_{thr}, J/cm^2	w^*, J/cm^2
Water	998	0,073	$1{,}00\cdot10^{-6}$	1080	2256	1,4	2,1
Ethanol	789	0,022	$1{,}52\cdot10^{-6}$	480	840	2,1	2,7
Glycerine	1260	0,065	$4{,}7\cdot10^{-5}$ (70 °C) $7{,}9\cdot10^{-4}$ (24 °C)	3640	962	1,6 (70 °C) 3,9 (24 °C)	2,2 (70 °C) —

This condition was satisfied in the experiments for all the fluids considered (their parameters are presented in Table 1 according to data from [13]), with the exception of glycerine at T = 24°C.

The theoretical analysis performed yields the following flow pattern. If

$$\Pi_0 < \sqrt{a\sigma\rho_0}, \tag{2.8}$$

then the GCW recoil pressure described by (2.1) that diverges from the center of the domain of application occurs on the fluid surface. When condition (2.6) is satisfied the motion is substantially nonlinear in nature: after a short transition stage, the surface being perturbed takes the shape of an expanding hemisphere, where the initial motion energy E_0 is a quantity on the order of 0.1 mJ, as follows from the experimental data presented below, while the free energy increment for values of the hemisphere radii considered in this paper ($R \lesssim 6$ mm) is just ~0.01 mJ, which indicates the validity of the assumption made in deriving (2.5). Therefore, cessation of the hemisphere expansion in our case is not associated with motion energy expenditure on the free energy increment of the fluid. A change in the nature of the motion occurs because of "flapping" of the fluid hemisphere which is displaced above the surface during expansion and having just a vertical velocity is not entrained in the horizontal direction behind the hemisphere but is interlocked above it to form a closed volume. In this situation, the relationship (2.3) is not valid since the work against the difference in the forces of the external pressure and the pressure within the expanding cavity should be taken additionally into account. A detailed hydrodynamic description of this kind of motion can be found in [14]. The computations performed are in good agreement with the experimental results, after flapping of the cavity the expansion ceases more rapidly than follows from (2.5).

Let us note that the deductions of the theoretical analysis about the existence of the dependence $R \sim t^{2/5}$ are in complete agreement with the data in [10] in which cylindrical and point explosions are investigated on a free fluid surface and the deduction is made that the fluid motion caused by the point explosion on its surface is almost self-similar with index $n \approx 0.38$.

3. EXPERIMENTAL RESULTS

An electrodischarge CO_2-laser whose radiation (10.6 μm wavelength) is absorbed strongly by all the fluids investigated (the appropriate absorption coefficients α are presented in Table 1 according to data in [15, 16]) was used for an experimental study of the process of optical radiation interaction with dielectric fluids. Visualization of the surface perturbations was performed by the usual shadow scheme (see [2], for example) with pulse bias lighting by an aluminum-yttrium garnet laser (30 nsec pulse duration, wavelength 0.53 μm after frequency doubling). The time delay between the times of CO_2-laser bias-lighting laser triggering was determined to an accuracy no worse than 1 μsec. The accuracy of measuring the vertical surface deflections is ~0.1 mm. The total pulse energy of CO_2-laser radiation varied between 0.05 and 0.6 J and was checked by a straight-through wire bolometer assembled in a differential circuit. Recording was realized in the 0.2-100 msec delay time range, which permitted a detailed study of the surface perturbation shape and of the dependence of the surface deflection amplitude on the laser radiation energy and on the time.

It was detected during the experiments that notable surface perturbations due to the action of the recoil pressure pulse occur only upon the achievement of a definite threshold value of the radiation energy density w_{thr} for each fluid. This is associated with the fact

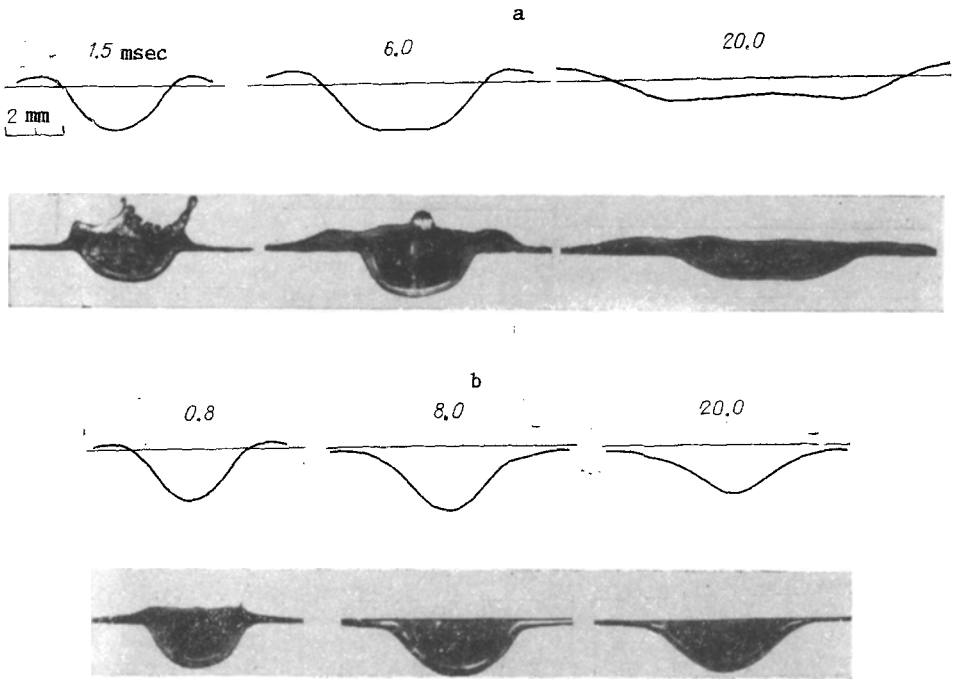

Fig. 2

that intensive evaporation processes do not occur for low energies of the radiation interacting with the fluid. Values of w_{thr} are indicated in Table 1, where appropriate values of the specific heat of vapor formation λ are also presented for comparison.

It should be noted that threshold energy densities whose excess results in an abrupt increase in the acoustic signal amplitude are also observed during recording of the acoustic signals being generated in the fluid under the action of the recoil pressure pulse. Such measurements, in particular, are performed in [6, 7] for the case of CO_2-laser radiation interaction with water. The value $w_{thr} \approx 1.4$ J/cm^2 that we found for water practically agrees with the magnitude of the threshold 1.5 J/cm^2 [7] and is somewhat below the value 2.5 J/cm^2 [6]. As was noted in [6], the presence of the threshold can be related to the passage from surface evaporation to a bulk phase transformation.

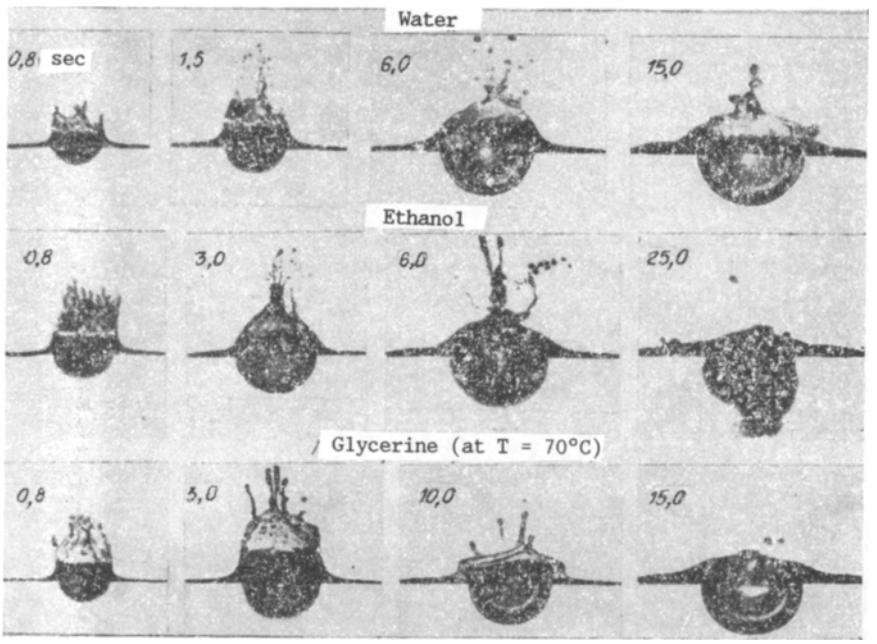

Fig. 3

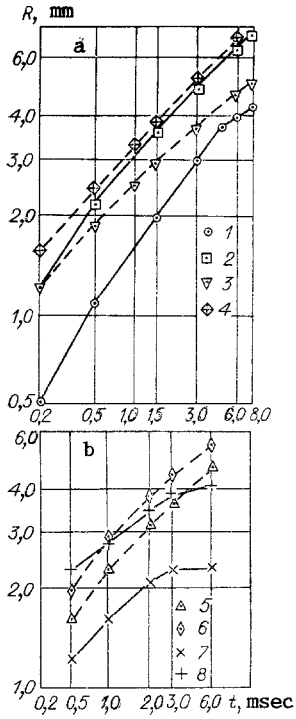

Fig. 4

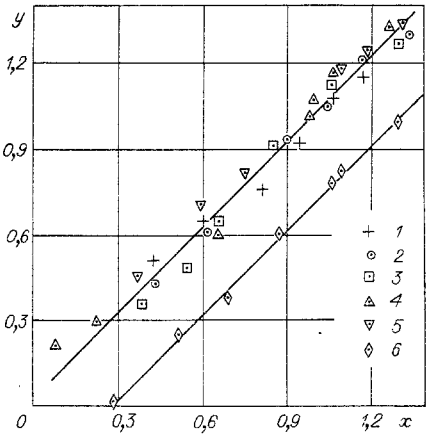

Fig. 5

The dependence of the surface deflection amplitude on the energy is such that for a 20-30% excess of w_{thr} the surface displacements occurring cannot be described within the framework of the linear approximation theory [condition (2.8)]. Systematic quantitative measurements were not performed in so narrow an energy range. However, it has been established that a good qualitative correspondence exists between the computed and the experimentally recorded surface profiles even for $h_{max} \sim a$. Thus, the water surface profiles in Fig. 2a for different times are compared with the computation performed by means of (2.1) under the assumption of a parabolic recoil pulse distribution $\Pi(r) = \Pi_0(1 - (r/a))^2$. The laser radiation energy density corresponding to the photographs in Fig. 2a is $w \simeq 2.0$ J/m^2 and the parameters used in the computation are: $\Pi_0 = 1$ Pa·sec, $a = 2.1$ mm [the condition for applicability of the theory of the linear approximation (2.8) requires $\Pi_0 < 0.4$ Pa·sec for this case]. An analogous comparison is performed in Fig. 2b for glycerine (at T = 24°C) under conditions when $w \simeq 4.3$ J/cm^2. The computation was executed by means of (2.2) for $\Pi_0 = 6.7$ Pa·sec, $a = 1.8$ mm.

The dynamics of the perturbation development for different fluid surfaces is represented in Fig. 3 for large values of the radiation energy. The laser radiation energy density w is 4.7 J/cm^2 for water, 5.8 J/cm^2 for ethanol, and 7.6 J/cm^2 for heated glycerine. The surface takes on the shape of a hemisphere expanding within the fluid for all three fluids in the investigated range of energy contributions at a definite stage.

In order to determine the expansion law, an experimental dependence of the hemisphere radius R on the time t is constructed in Fig. 4 in specially chosen coordinates. Plotted along the ordinate axis is ln R and along the abscissa (2/5)ln t. The experimental points correspond to the following energy contributions w: a — water: 1, 2) 2.8, 5.7 J/cm^2; ethanol: 3, 4) 4.25, 5.7 J/cm^2; b — glycerine (at T = 70°C): 5, 6) 4.2, 6.5 J/cm^2. Presented here for comparison are analogous data for glycerine at T = 24°C also: 7, 8) 6.6, 11.1 J/cm^2 (in this case it is necessary to speak about the amplitude of surface deflection rather than of the hemisphere radius since the shape of the surface is similar to that displayed in Fig. 2b for the mentioned energy contributions). It follows from Fig. 4 that for all energies, starting with a certain minimal value, a time interval exists in which the dependence R(t) is expressed by a power law with the exponent n = 2/5 in complete conformity with (2.7).

The dependence of the total recoil pressure pulse Π_0 on the radiation energy ε is of special interest for the construction of an adequate theoretical model of the process of optical radiation interaction with strongly absorptive dielectric fluids. The quantity Π_0 is not successfully measured directly in fluids. However, considering the relation between R and Π_0 given by (2.7) to be confirmed experimentally and determining $R(\varepsilon)$ from experiment,

it is easy to find the relationship between Π_0 and ε. Let us arrange experimental values of R obtained for times lying in the interval where the law $R \sim t^{2/5}$ is satisfied, in the coordinates in the graph of Fig. 5 selected with (2.7) taken into account as follows

$$y = \ln\left\{\left(\frac{\rho_0}{\sqrt{a}\,t}\right)^{2/5} RA\right\}, \quad x = \frac{2}{5}\ln\left\{\frac{\varepsilon - \varepsilon^*}{B}\right\}. \tag{3.1}$$

All the quantities in (3.1) are expressed in the SI system of units. The quantity ε^* is determined in experiment and corresponds to the energy at which the maximal deflection of the fluid surface from equilibrium equals the characteristic radius of the domain of recoil pressure action $R|_{t=t^*}(\varepsilon^*) = a$ [t^* is the time of reaching the maximal deflection calculated from the linear approximation formula (2.1)]. Values of the arbitrary constants A and B permitting the arguments of the logarithmic functions in (3.1) to be made dimensionless, are selected in Fig. 5 for convenience in the representation of the experimental data in the form $A = 1.11\ (\text{sec}\cdot\text{m/kg})^{2/5}$, $B = 1.7\cdot10^{-2}$ J. The following values of t correspond to different points: water: 1, 2) 1.5, 3 msec; ethanol: 3, 4) 1.5, 3 msec; glycerine (at T = 70°C): 5) 3 msec. For all cases a = 1.8 mm. Analogous measurements were also performed for other values of the radius. Presented as an example in Fig. 5 are data for water at a = 2.75 mm, t = 3 msec (point 6). As is seen from Fig. 5, the experimental points for a fixed laser beam radius lie on the very same line for all the fluids studied. The expression for the dependence of R on ε:

$$R = q(b)\left[\frac{\sqrt{a}\,t}{\rho_0}(\varepsilon - \varepsilon^*)\right]^{2/5} \tag{3.2}$$

gives an approximation of this line [q(b) is a general proportionality factor dependent on the laser beam radius b]. Also a function of the radius b is ε^*. Analysis of the experimental data permits establishment that the factor is $q \sim 1/b^{4/5}$ and $\varepsilon^* \sim b^2$ in the investigated range of variations of the beam radius 1.2-2.5 mm. Now comparing (3.2) with the theoretical formula (2.7) we write the approximate relationship

$$\Pi_0 \simeq \gamma(w - w^*). \tag{3.3}$$

Here $w = \varepsilon/(\pi b^2)$ and the values of the constants $w^* = \varepsilon^*/(\pi b^2)$ are indicated in Table 1 for each of the fluids. Let us note that (3.3) is valid for $w - w^* \gg \sqrt{a\sigma\rho_0}/\gamma$ [in conformity with condition (2.6)]. The factor γ is identical for water, ethanol, and glycerine heated to 70°C: $\gamma = 2.4\cdot10^{-4}$ sec/m. This circumstance should be taken into account in the construction of a theory of volume evaporation called to give a description of the process of intensive laser radiation interaction with strongly absorptive dielectric fluids. According to (3.3), the values of the total recoil pressure pulses obtained in our experiments for water lie in the range $\Pi_0 \approx$ (2.4-15.1) Pa·sec for w = (3.1-8.4) J/cm^2. We mention, for comparison that according to the data of acoustic measurements performed in [7] for an analogous experimental situation (the interaction of radiation of an electro-discharge CO_2-laser with water) for larger laser beam radii (~5 mm), the total recoil pressure pulse was $\Pi_0 \sim 1$ Pa·sec for $w \approx 0.85$ J/cm^2. The growth of the ratio Π_0/w as the spot radius increases and unchanged w is associated with the rise in the efficiency of recoil reaction application under conditions of more prolonged conservation of the plane geometry of the vapor escape.

Let us note that the regularities established experimentally in this paper permit estimation of the recoil pressure pulse transmitted to the fluid surface according to the given energy of laser radiation. The explanation of these regularities requires further development of the theory of the phenomenon.

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FLOW OF A VISCOUS LIQUID IN A LAYER ON A ROTATING PLANE

O. M. Lavrent'eva

UDC 532

In carrying out certain modern industrial processes, the application of thin films of uniform thickness onto a flat surface is required. One of the methods used to accomplish this consists in first pouring a sufficiently thick layer of the liquid onto the surface, which then thins out as the specimen is rotated [1]. Similar methods are used in making mirrors [2], color television screens [3], integral schemes, and magnetic memory disks [1]. Rotating disks are also used for spraying and for mixing liquids to accelerate heterogeneous chemical reactions in various processes of chemical technology [4-6].

To effectively control these processes one needs to know the nature of the flows that arise. Since the radius of the rotating disk is usually many times the thickness of the liquid layer, one can, for the purposes of mathemetical modelling, replace the disk by an infinite rotating plane. In the present paper we construct stationary and self-similar Karman-type solutions of the Navier–Stokes equations, which describe the flow of a viscous liquid in the layer between a rotating solid plane and the free surface parallel to it.

1. Statement of the Problem. We consider a rotationally-symmetric flow of a viscous incompressible liquid in a layer $\Lambda_t = \{(r, \theta, z) \in R^3, z \in (0, Z(t))\}$, bounded above by its free surface and below by a solid wall rotating around the z-axis at a given angular rate $\Omega(t)$.

The field of velocity and pressure in the liquid [V(r, z, t) and p(r, z, t)] satisfies the Navier–Stokes equations

$$\begin{gathered} u_t + uu_r - r^{-1}v^2 + wu_z = -\rho^{-1}p_r + \nu[u_{rr} + (r^{-1}u)_r + u_{zz}], \\ v_t + uv_r + r^{-1}uv + wv_z = \nu[v_{rr} + (r^{-1}v)_r + v_{zz}], \\ w_t + uw_r + ww_z = -\rho^{-1}p_z + \nu[w_{rr} + r^{-1}w_r + w_{zz}], \\ u_r + r^{-1}u + w_z = 0 \end{gathered} \tag{1.1}$$

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